Fundamental Laws of Electrostatics

- Integral form

$$\oint_C \underline{E} \cdot d\underline{l} = 0$$

$$\oint_S \underline{D} \cdot d\underline{s} = \int_V q_{ev}\, dv$$

- Differential form

$$\nabla \times \underline{E} = 0$$

$$\nabla \cdot \underline{D} = q_{ev}$$

$$\overline{D} = \varepsilon \overline{E}$$

Fundamental Laws of Magnetostatics

- Integral form
- Differential form

$$\oint_C \underline{H} \cdot d\underline{l} = \int_S \underline{J} \cdot d\underline{s}$$

$$\oint_S \underline{B} \cdot d\underline{s} = 0$$

$$\nabla \times \underline{H} = \underline{J}$$

$$\nabla \cdot \underline{B} = 0$$

$$\underline{B} = \mu \underline{H}$$

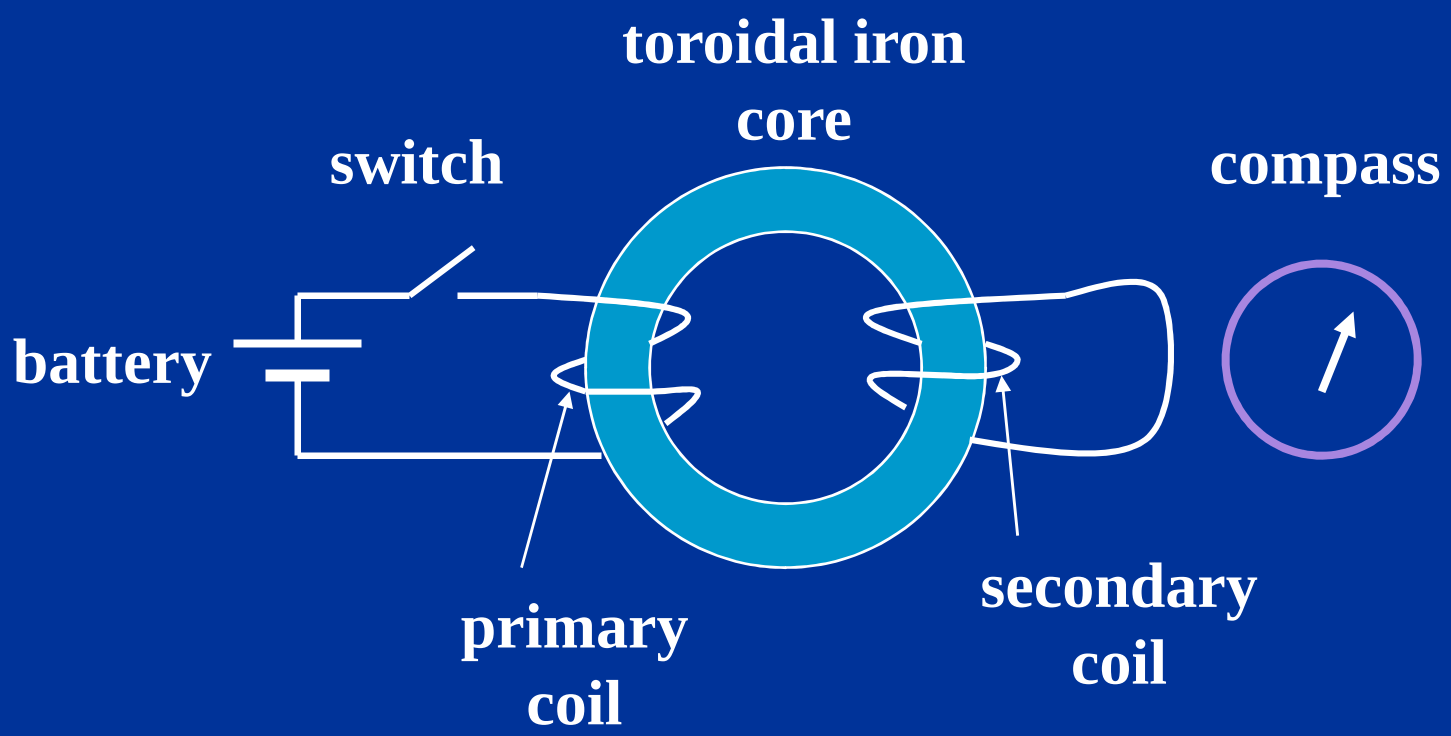
Electrostatic, Magnetostatic, and Electromagnetostatic Fields

- In the static case (no time variation), the electric field (specified by $\boldsymbol{E}$ and $\boldsymbol{D}$) and the magnetic field (specified by $\boldsymbol{B}$ and $\boldsymbol{H}$) are described by separate and independent sets of equations.
- In a conducting medium, both electrostatic and magnetostatic fields can exist, and are coupled through the Ohm's law ($\boldsymbol{J} = \sigma \boldsymbol{E}$). Such a situation is called ***electromagnetostatic***.

The Three Experimental Pillars of Electromagnetics

- Electric charges attract/repel each other as described by *Coulomb's law*.
- Current-carrying wires attract/repel each other as described by *Ampere's law of force*.
- Magnetic fields that change with time induce electromotive force as described by ***Faraday's law***.

Faraday's Experiment



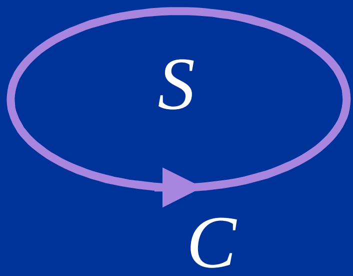
Faraday's Experiment (Cont'd)

- Upon closing the switch, current begins to flow in the *primary coil*.
- A momentary deflection of the *compass needle* indicates a brief surge of current flowing in the *secondary coil*.
- The *compass needle* quickly settles back to zero.
- Upon opening the switch, another brief deflection of the *compass needle* is observed.

Faraday's Law of Electromagnetic Induction

- "The electromotive force induced around a closed loop ***C*** is equal to the time rate of decrease of the magnetic flux linking the loop."

$$V_{ind} = -\frac{d\Phi}{dt}$$



Faraday's Law of Electromagnetic Induction (Cont'd)

$$\Phi = \int_S \underline{B} \cdot d\,\underline{s}$$

$$V_{ind} = \oint_C \underline{E} \cdot d\,\underline{l}$$

• S is any surface bounded by C

$$\boxed{\oint_C \underline{E} \cdot d\,\underline{l} = -\frac{d}{dt}\int_S \underline{B} \cdot d\,\underline{s}}$$

integral form of Faraday's law

Faraday's Law (Cont'd)

Stokes's theorem

$$\oint_C \underline{E} \cdot d\underline{l} = \int_S \nabla \times \underline{E} \cdot d\underline{s}$$

$$-\frac{d}{dt}\int_S \underline{B} \cdot d\underline{s} = -\int_S \frac{\partial \underline{B}}{\partial t} \cdot d\underline{s}$$

assuming a stationary surface *S*

Faraday's Law (Cont'd)

- Since the above must hold for any *S*, we have

$$\boxed{\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}}$$

differential form of Faraday's law (assuming a stationary frame of reference)

Faraday's Law (Cont'd)

- Faraday's law states that a changing magnetic field induces an electric field.
- The induced electric field is *non-conservative*.

Lenz's Law

- "The sense of the emf induced by the time-varying magnetic flux is such that any current it produces tends to set up a magnetic field that opposes the <u>change</u> in the original magnetic field."
- Lenz's law is a consequence of conservation of energy.
- Lenz's law explains the minus sign in Faraday's law.

Faraday's Law

- "The electromotive force induced around a closed loop *C* is equal to the time rate of decrease of the magnetic flux linking the loop."

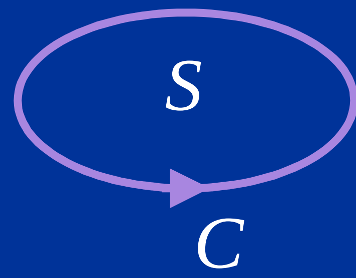
$$V_{ind} = -\frac{d\Phi}{dt}$$

- For a coil of N tightly wound turns

$$V_{ind} = -N\frac{d\Phi}{dt}$$

Faraday's Law (Cont'd)

$$\Phi = \int_S \underline{B} \cdot d\,\underline{s}$$



$$V_{ind} = \oint_C \underline{E} \cdot d\,\underline{l}$$

- *S* is any surface bounded by *C*

Faraday's Law (Cont'd)

- Faraday's law applies to situations where
  - (1) the *B*-field is a function of time
  - (2) ***ds*** is a function of time
  - (3) ***B*** and ***ds*** are functions of time

Ampere's Law and the Continuity Equation

- The differential form of Ampere's law in the static case is

$$\nabla \times \underline{H} = \underline{J}$$

- The continuity equation is

$$\nabla \cdot \underline{J} + \frac{\partial q_{ev}}{\partial t} = 0$$

Ampere's Law and the Continuity Equation (Cont'd)

- In the time-varying case, Ampere's law in the above form is inconsistent with the continuity equation

$$\nabla \cdot \underline{J} = \nabla \cdot \left(\nabla \times \underline{H} \right) = 0$$

Ampere's Law and the Continuity Equation (Cont'd)

- To resolve this inconsistency, Maxwell modified Ampere's law to read

$$\nabla \times \underline{H} = \underline{J}_c + \frac{\partial \underline{D}}{\partial t}$$

conduction current density

displacement current density

Ampere's Law and the Continuity Equation (Cont'd)

- The new form of Ampere's law is consistent with the continuity equation as well as with the differential form of Gauss's law

$$\nabla \cdot \underline{J}_c + \frac{\partial}{\partial t}\left(\underbrace{\nabla \cdot \underline{D}}_{q_{ev}}\right) = \nabla \cdot \left(\nabla \times \underline{H}\right) = 0$$

Displacement Current

- Ampere's law can be written as

$$\boxed{\nabla \times \underline{H} = \underline{J}_c + \underline{J}_d}$$

where

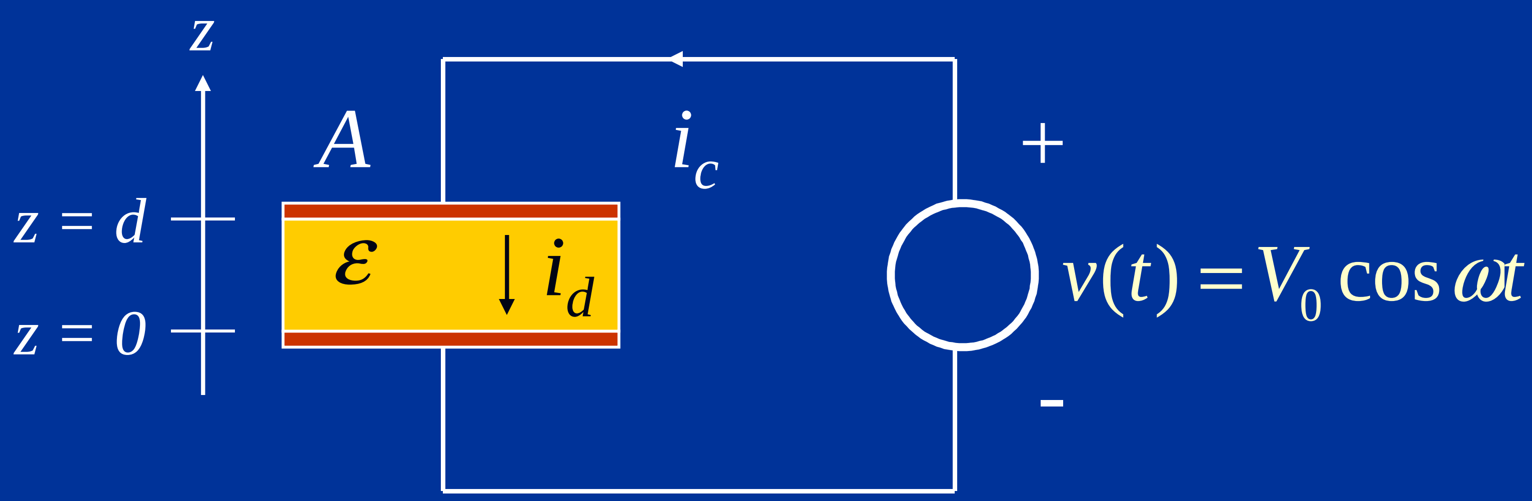
$$\underline{J}_d = \frac{\partial \underline{D}}{\partial t} = \text{displacement current density (A/m}^2\text{)}$$

Displacement Current (Cont'd)

- ***Displacement current*** is the type of current that flows between the plates of a capacitor.
- *Displacement current* is the mechanism which allows electromagnetic waves to propagate in a non-conducting medium.
- *Displacement current* is a consequence of the three experimental pillars of electromagnetics.

Displacement Current in a Capacitor

- Consider a parallel-plate capacitor with plates of area A separated by a dielectric of permittivity ε and thickness d and connected to an *ac* generator:



Displacement Current in a Capacitor (Cont'd)

- The electric field and displacement flux density in the capacitor is given by

$$\underline{E} = -\hat{a}_z \frac{v(t)}{d} = -\hat{a}_z \frac{V_0}{d}\cos\omega t$$

$$\underline{D} = \varepsilon \underline{E} = -\hat{a}_z \frac{\varepsilon V_0}{d}\cos\omega t$$

• assume fringing is negligible

- The displacement current density is given by

$$\underline{J}_d = \frac{\partial \underline{D}}{\partial t} = \hat{a}_z \frac{\omega\varepsilon V_0}{d}\sin\omega t$$

Displacement Current in a Capacitor (Cont'd)

- The displacement current is given by

$$i_d = \int_S \underline{J}_d \cdot d\,\underline{s} = -J_d A = -\frac{\omega \varepsilon A}{d} V_0 \sin \omega t$$

$$= -\omega C V_0 \sin \omega t = C \frac{dv}{dt} = i_c$$

conduction current in wire

Maxwell's Equations in Differential Form for Time-Harmonic Fields in Simple Medium

$$\nabla \times \underline{E} = -\left(j\omega\mu + \sigma_m \right) \underline{H} - \underline{K}_i$$

$$\nabla \times \underline{H} = \left(j\omega\varepsilon + \sigma_e \right) \underline{E} + \underline{J}_i$$

$$\nabla \cdot \underline{E} = \frac{q_{ev}}{\varepsilon}$$

$$\nabla \cdot \underline{H} = \frac{q_{mv}}{\mu}$$

Maxwell's Curl Equations for Time-Harmonic Fields in Simple Medium Using Complex Permittivity and Permeability

complex
permeability

$$\nabla \times \underline{E} = -j\omega\mu\underline{H} - \underline{K}_i$$

$$\nabla \times \underline{H} = j\omega\varepsilon\underline{E} + \underline{J}_i$$

complex
permittivity

Overview of Waves

- A ***wave*** is a pattern of values in space that appear to move as time evolves.
- A *wave* is a solution to a *wave equation*.
- Examples of waves include water waves, sound waves, seismic waves, and voltage and current waves on transmission lines.

Overview of Waves (Cont'd)

- Wave phenomena result from an exchange between two different forms of energy such that the time rate of change in one form leads to a spatial change in the other.
- Waves possess
  - no mass
  - energy
  - momentum
  - velocity

Time-Domain Maxwell's Equations in Differential Form

$$\underline{K}_c + \underline{K}_i$$

$$\nabla \times \underline{E} = -\underline{K} - \frac{\partial \underline{B}}{\partial t} \qquad \nabla \cdot \underline{D} = q_{ev}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \qquad \nabla \cdot \underline{B} = q_{mv}$$

$$\overline{J}_c + \overline{J}_i$$

Time-Domain Maxwell's Equations in Differential Form for a Simple Medium

$$\underline{D} = \varepsilon \underline{E} \qquad \underline{B} = \mu \underline{H} \qquad \underline{J}_c = \sigma \underline{E} \qquad \underline{K}_c = \sigma_m \underline{H}$$

$$\boxed{\begin{array}{ll} \nabla \times \underline{E} = -\sigma_m \underline{H} + \underline{K}_i - \mu \dfrac{\partial \underline{H}}{\partial t} & \nabla \cdot \underline{E} = \dfrac{q_{ev}}{\varepsilon} \\ \nabla \times \underline{H} = \sigma \underline{E} + \underline{J}_i + \varepsilon \dfrac{\partial \underline{E}}{\partial t} & \nabla \cdot \underline{H} = \dfrac{q_{mv}}{\varepsilon} \end{array}}$$

Time-Domain Maxwell's Equations in Differential Form for a Simple, Source-Free, and Lossless Medium

$$\underline{J}_i = \underline{K}_i = 0 \quad q_{ev} = q_{mv} = 0 \quad \sigma = \sigma_m = 0$$

$$\boxed{\begin{array}{ll} \nabla \times \underline{E} = -\mu \dfrac{\partial \underline{H}}{\partial t} & \nabla \cdot \underline{E} = 0 \\ \nabla \times \underline{H} = \varepsilon \dfrac{\partial \underline{E}}{\partial t} & \nabla \cdot \underline{H} = 0 \end{array}}$$

Time-Domain Maxwell's Equations in Differential Form for a Simple, Source-Free, and Lossless Medium

- Obviously, there must be a source for the field somewhere.
- However, we are looking at the properties of waves in a region far from the source.

Derivation of Wave Equations for Electromagnetic Waves in a Simple, Source-Free, Lossless Medium

$$\nabla\times\nabla\times\underline{E} = \nabla\left(\overbrace{\nabla\cdot\underline{E}}^{0}\right) - \nabla^2\underline{E}$$

$$= -\mu\frac{\partial\left(\nabla\times\underline{H}\right)}{\partial t} = -\mu\varepsilon\frac{\partial^2\underline{E}}{\partial t^2}$$

$$\nabla\times\nabla\times\underline{H} = \nabla\left(\overbrace{\nabla\cdot\underline{H}}^{0}\right) - \nabla^2\underline{H}$$

$$= \varepsilon\frac{\partial\left(\nabla\times\underline{E}\right)}{\partial t} = -\mu\varepsilon\frac{\partial^2\underline{H}}{\partial t^2}$$

Wave Equations for Electromagnetic Waves in a Simple, Source-Free, Lossless Medium

$$\boxed{\nabla^2 \underline{E} - \mu\varepsilon \frac{\partial^2 \underline{E}}{\partial t^2} = 0}$$

$$\boxed{\nabla^2 \underline{H} - \mu\varepsilon \frac{\partial^2 \underline{H}}{\partial t^2} = 0}$$

- The wave equations are not independent.
- Usually we solve the electric field wave equation and determine ***H*** from ***E*** using Faraday's law.

Uniform Plane Wave Solutions in the Time Domain

- A ***uniform plane wave*** is an electromagnetic wave in which the electric and magnetic fields and the direction of propagation are mutually orthogonal, and their amplitudes and phases are constant over planes perpendicular to the direction of propagation.
- Let us examine a possible plane wave solution given by

$$\underline{E} = \hat{a}_x \, E_x(z,t)$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The wave equation for this field simplifies to

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0}$$

- The general solution to this wave equation is

$$E_x(z,t) = p_1(z - v_p t) + p_2(z + v_p t)$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The functions $p_1(z-v_p t)$ and $p_2(z+v_p t)$ represent uniform waves propagating in the +z and -z directions respectively.
- Once the electric field has been determined from the wave equation, the magnetic field must follow from Maxwell's equations.

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The ***velocity of propagation*** is determined solely by the medium:

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

- The functions p_1 and p_2 are determined by the source and the other boundary conditions.

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- Here we must have

$$\underline{H} = \hat{a}_y\, H_y(z,t)$$

where

$$H_y(z,t) = \frac{1}{\eta}\left\{p_1\left(z - v_p t\right) - p_2\left(z + v_p t\right)\right\}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- η is the ***intrinsic impedance*** of the medium given by

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

- Like the velocity of propagation, the intrinsic impedance is independent of the source and is determined only by the properties of the medium.

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- In free space (vacuum):

$$\boxed{\begin{aligned} v_p &= c \approx 3\times10^8 \text{ m/s} \\ \eta &= 120\pi \approx 377\Omega \end{aligned}}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- Strictly speaking, uniform plane waves can be produced only by sources of infinite extent.
- However, point sources create spherical waves. Locally, a spherical wave looks like a plane wave.
- Thus, an understanding of plane waves is very important in the study of electromagnetics.

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- Assuming that the source is sinusoidal. We have

$$p_1\left(z - v_p t\right) = C_1 \cos\left(\frac{\omega}{v_p}\left(z - v_p t\right)\right) = C_1 \cos\left(\omega t - \beta z\right)$$

$$p_2\left(z + v_p t\right) = C_2 \cos\left(\frac{\omega}{v_p}\left(z - v_p t\right)\right) = C_2 \cos\left(\omega t + \beta z\right)$$

$$\beta = \frac{\omega}{v_p}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The electric and magnetic fields are given by

$$E_x(z,t) = C_1 \cos(\omega t - \beta z) + C_2 \cos(\omega t + \beta z)$$

$$H_y(z,t) = \frac{1}{\eta}\{C_1 \cos(\omega t - \beta z) - C_2 \cos(\omega t + \beta z)\}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The argument of the cosine function is the called the ***instantaneous phase*** of the field:

$$\phi(z,t) = \omega t - \beta z$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The speed with which a constant value of instantaneous phase travels is called the ***phase velocity***. For a lossless medium, it is equal to and denoted by the same symbol as the *velocity of propagation*.

$$\omega t - \beta z = \phi_0 \Rightarrow z = \frac{\omega t - \phi_0}{\beta}$$

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

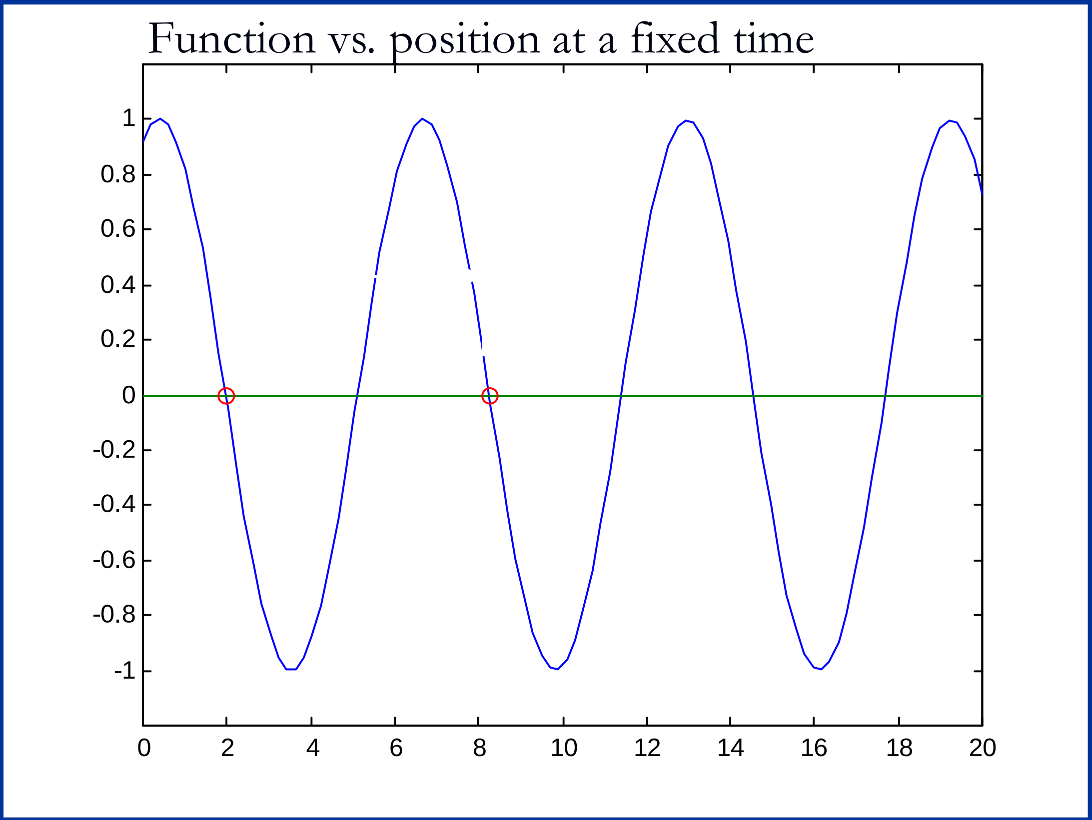
Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The distance along the direction of propagation over which the instantaneous phase changes by 2π radians for a fixed value of time is the ***wavelength***.

$$\beta\lambda = 2\pi \Rightarrow \lambda = \frac{2\pi}{\beta}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- The ***wavelength*** is also the distance between every other zero crossing of the sinusoid.



Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- Relationship between *wavelength* and frequency in free space:

$$\lambda = \frac{c}{f}$$

- Relationship between *wavelength* and frequency in a material medium:

$$\boxed{\lambda = \frac{v_p}{f}}$$

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- β is the ***phase constant*** and is given by

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v_p}$$

rad/m

Uniform Plane Wave Solutions in the Time Domain (Cont'd)

- In free space (vacuum):

$$\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c} = k_0 = \frac{2\pi}{\lambda_0}$$

free space wavenumber
(rad/m)

Flow of Electromagnetic Power

- Electromagnetic waves transport throughout space the energy and momentum arising from a set of charges and currents (the sources).
- If the electromagnetic waves interact with another set of charges and currents in a receiver, information (energy) can be delivered from the sources to another location in space.
- The energy and momentum exchange between waves and charges and currents is described by the Lorentz force equation.

Derivation of Poynting's Theorem

- Poynting's theorem concerns the conservation of energy for a given volume in space.
- Poynting's theorem is a consequence of Maxwell's equations.

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- Time-Domain Maxwell's curl equations in differential form

$$\nabla \times \underline{E} = -\underline{K}_i - \underline{K}_c - \frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{H} = \underline{J}_i + \underline{J}_c + \frac{\partial \underline{D}}{\partial t}$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- Recall a vector identity

$$\nabla \cdot \left(\underline{E} \times \underline{H} \right) = \underline{H} \cdot \nabla \times \underline{E} - \underline{E} \cdot \nabla \times \underline{H}$$

- Furthermore,

$$-\underline{E} \cdot \nabla \times \underline{H} = -\underline{E} \cdot \underline{J}_i - \underline{E} \cdot \underline{J}_c - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t}$$

$$\underline{H} \cdot \nabla \times \underline{E} = -\underline{H} \cdot \underline{K}_i - \underline{H} \cdot \underline{K}_c - \underline{H} \cdot \frac{\partial \underline{B}}{\partial t}$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

$$\nabla\cdot\left(\underline{E}\times\underline{H}\right)=\underline{H}\cdot\nabla\times\underline{E}-\underline{E}\cdot\nabla\times\underline{H}$$

$$=-\underline{H}\cdot\underline{K}_i-\underline{H}\cdot\underline{K}_c-\underline{H}\cdot\frac{\partial\underline{B}}{\partial t}$$

$$-\underline{E}\cdot\underline{J}_i-\underline{E}\cdot\underline{J}_c-\underline{E}\cdot\frac{\partial\underline{D}}{\partial t}$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- Integrating over a volume V bounded by a closed surface S, we have

$$\int_V \left(\underline{E}\cdot\underline{J}_i + \underline{H}\cdot\underline{K}_i\right)dv = -\int_V \left(\underline{E}\cdot\frac{\partial \underline{D}}{\partial t} + \underline{H}\cdot\frac{\partial \underline{B}}{\partial t}\right)dv - \int_V \underline{E}\cdot\underline{J}_c\,dv$$

$$-\int_V \underline{H}\cdot\underline{M}_c\,dv - \int_V \nabla\cdot\left(\underline{E}\times\underline{H}\right)dv$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- Using the divergence theorem, we obtain the general form of Poynting's theorem

$$\boxed{\begin{aligned}\int_V \left(\underline{E}\cdot\underline{J}_i+\underline{H}\cdot\underline{K}_i\right)dv = &-\int_V\left(\underline{E}\cdot\frac{\partial\underline{D}}{\partial t}+\underline{H}\cdot\frac{\partial\underline{B}}{\partial t}\right)dv-\int_V\underline{E}\cdot\underline{J}_c\,dv\\ &-\int_V\underline{H}\cdot\underline{M}_c\,dv-\oint_S\left(\underline{E}\times\underline{H}\right)\cdot d\underline{s}\end{aligned}}$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- For simple, lossless media, we have

$$\int_V \left(\underline{E}\cdot\underline{J}_i + \underline{H}\cdot\underline{K}_i\right)dv = -\int_V \left(\varepsilon\underline{E}\cdot\frac{\partial\underline{E}}{\partial t} + \mu\underline{H}\cdot\frac{\partial\underline{H}}{\partial t}\right)dv$$

$$-\oint_S \left(\underline{E}\times\underline{H}\right)\cdot d\,\underline{s}$$

- Note that

$$\underline{A}\cdot\frac{\partial\underline{A}}{\partial t} = A\frac{\partial A}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\left(A^2\right)$$

Derivation of Poynting's Theorem in the Time Domain (Cont'd)

- Hence, we have the form of Poynting's theorem valid in simple, lossless media:

$$\boxed{\begin{aligned}\int_V \left(\underline{E}\cdot\underline{J}_i + \underline{H}\cdot\underline{K}_i\right)dv = -\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right)dv \\ -\oint_S \left(\underline{E}\times\underline{H}\right)\cdot d\,\underline{s}\end{aligned}}$$

Physical Interpretation of the Terms in Poynting's Theorem

- The terms

$$\int_V \sigma E^2 dv + \int_V \sigma_m H^2 dv$$

represent the ***instantaneous power dissipated*** in the electric and magnetic conductivity losses, respectively, in volume *V*.

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- The terms

$$\int_V \omega\varepsilon'' E^2 dv + \int_V \omega\mu'' H^2 dv$$

represent the ***instantaneous power dissipated*** in the polarization and magnetization losses, respectively, in volume V.

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- Recall that the electric energy density is given by

$$w_e = \frac{1}{2}\varepsilon' E^2$$

- Recall that the magnetic energy density is given by

$$w_m = \frac{1}{2}\mu' H^2$$

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- Hence, the terms

$$\int_V \left(\frac{1}{2}\varepsilon' E^2 + \frac{1}{2}\mu' H^2 \right) dv$$

represent the ***total electromagnetic energy stored*** in the volume V.

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- The term

$$\oint_S \left(\overline{E} \times \overline{H}\right) \cdot d\overline{s}$$

represents ***the flow of instantaneous power*** out of the volume V through the surface S.

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- The term

$$\int_V \left(\underline{E} \cdot \underline{J}_i + \underline{H} \cdot \underline{K}_i \right) dv$$

represents the *total electromagnetic energy generated by the sources* in the volume V.

Physical Interpretation of the Terms in Poynting's Theorem (Cont'd)

- **In words the Poynting vector can be stated as** ***"The sum of the power generated by the sources, the imaginary power (representing the time-rate of increase) of the stored electric and magnetic energies, the power leaving, and the power dissipated in the enclosed volume is equal to zero."***

$$\boxed{\begin{aligned}&\int_V \left(\underline{E}\cdot\underline{J}_i+\underline{H}\cdot\underline{K}_i\right)dv+j\omega\int_V\left(\frac{1}{2}\varepsilon' E^2+\frac{1}{2}\mu' H^2\right)dv+\omega\int_V\left(\varepsilon'' E^2+\mu'' H^2\right)dv\\&+\int_V \sigma E^2\,dv+\int_V \sigma_m H^2 dv+\oint_S\left(\underline{E}\times\underline{H}\right)\cdot d\,\underline{s}=0\end{aligned}}$$

Poynting Vector in the Time Domain

- We define a new vector called the (instantaneous) ***Poynting vector*** as

$$\overline{S} = \overline{E} \times \overline{H}$$

• The Poynting vector has units of W/m^2.

- The Poynting vector has the same direction as the direction of propagation.
- The Poynting vector at a point is equivalent to the power density of the wave at that point.